Review, 4-26 and 4-28

- Bayesian verse Frequentist Learning
- Why / when to use Dev within folds of N-Fold CV
- Time series -- what distinguishes
- Causal Inference
- Autocorrelation
    - Type of univariate time series
    - Lag Plots

Autoregressive Model

AR Models: $$Y_t = f(Y_{t-1}, Y_{t-2}, Y_{t-3}, \dots, Y_{t-n}, \epsilon_t)$$

Linear AR model: $$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \dots + \beta_n Y_{t-p} + \epsilon_t$$

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Notation:

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Based on error;   (a “smoothing” technique).

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Simple Moving Average

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attributed to “shocks” -- independent, from a normal distribution

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ARMA Models

AutoRegressive (AR) Moving Average (MA) Model

ARMA(p, q):

$$\hat{Y}_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + ... + \beta_p Y_{t-p} +$$
$$\epsilon_t + \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + ... + \theta_q \epsilon_{t-q}$$

ARMA(1, 1):

$$\hat{Y}_t = \beta_1 Y_{t-1} + \epsilon_t + \theta_1 \epsilon_{t-1}$$

example: Y is sales; error may be effect from coupon or advertising

(credit: Ben Lambert)

Time-series Applications

- ARMA
  - Economic indicators
  - System performance
  - Trend analysis
    (often situations where there is a general trend and random “shocks”)
- Univariate Models in General
  - Anomaly Detection
  - Forecasting
  - Season Trends
  - Signal Processing
- Integration as predictors within multivariate models

`statsmodels.tsa.arima_model`